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A Distributed Online Heuristic for a Large-scale Workforce Task Assignment and Multi-Vehicle Routing Problem

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Outline

1. Problem statement and contributions
2. MILP formulation
3. Online gossip-based heuristic
4. Numerical simulation on real data
5. Conclusions and future work

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Problem of interest (the real case study of DEDEM S.p.A)

Problem: Managing a workforce of technicians responsible for refurbishing and repairing photo booths across the Italian territory.

Objective: Maximize the profit for the enterprise

Three-folded challenge:

- Task assignment
- Route planning
- Execution scheduling

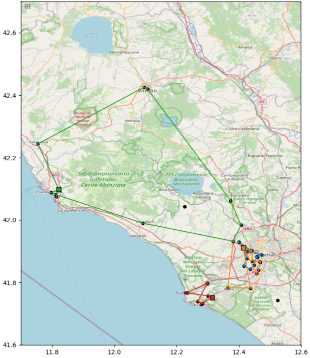
Problem of interest (the real case study of DEDEM S.p.A)

Main differences with the standard MVRP (multi-veichle routing problem):

1. Each photo-boot (a task) generates an hourly profit (€/hour) once it has been refurbished/repaired:
   - Tasks may not be executed
2. Each technician (a vehicle) has a daily cost (€/day):
   - Technicians may not be employed

Additional complexities:

- Time windows for technicians and tasks
- Skills required for the tasks' execution
- Multi-day assignment/routing/scheduling
- Calendar of technicians' availability



DEDEM

Problem of interest (the real case study of DEDEM S.p.A)

Our main contributions are:

- A MILP formulation of the complex optimization scenario of the DEDEM S.p.A. case study
- A gossip-based heuristic algorithm with the following features:
  - Scalability for large problem sizes
  - Real-time employment to deal with:
    - Delays on task execution and traveling times
    - New tasks available to be performed
    - Unexpected unavailability of the technicians
  - Geolocation privacy-preserving
- Numerical tests validating the approach by using real data drawn from DEDEM S.p.A.'s data

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Decision variables

We consider an undirected graph $G = (V, E)$ defined as follows:

- The set of nodes V can be decomposed into three disjoint sets:
  - $S = \{1, \ldots, s\}$ is the set of technicians' depots and s is the total number of technicians;
  - $D = \cup_{\ell=1}^{d-1} D_\ell$ where $D_\ell = \{\ell s + 1, \cdots, (l+1)s\}$ is the set of the technicians' depots corresponding to the ℓ-th night and d is the total number of days;
  - $P = \{ds, \ldots, ds + p\}$ is the set of tasks' position and p is the total number of tasks.
- The set of edges E representing the viable paths is such that:

$$E : \begin{cases} (i,j),\ (j,i) \in E & i, j \in P, \\ (i,j),\ (j,i) \in E & i \in S \cup D, j \in P, \\ ((\ell-1)s + i, \ell s + i) \in E & i \in S,\ \ell = 1, \ldots, d-1, \\ (i,j) \notin E & \text{otherwise}, \end{cases}$$

We adapt the multi-commodity flow formulation[1] to our case study and consider the variables:

- $X_{i,j} \in \{0,1\}$ denotes the motion of a technician through the $(i,j) \in E$.
- $U_{i,j} \in [0, \infty)$ denotes the initial time of motion from $i \in V$ to $j \in V$.
- $F_{i,j,k} \in \{0,1\}$ denotes the motion of a technician from task k through $(i,j) \in E$.

Objective function

$$f(X,F,U) = \mathrm{FUEL}(X) + \mathrm{TECH}(F) - \mathrm{TASKS}(X,U).$$

with

$$\mathrm{FUEL}(X) = \sum_{(i,j)\in E} f_{ij} d_{ij} X_{i,j}$$

$$\mathrm{TECH}(F) = \sum_{i\in S} q_i \sum_{j\in P} F_{i,j,i} + \sum_{\ell\in 1}^{d-1} \sum_{i\in D_\ell} q_{i-\ell s} \sum_{j\in P} F_{i,j,i}$$

$$\mathrm{TASKS}(X,U) = \sum_{j\in P} g_j \sum_{i\in V} \left[(24\cdot d - t_{ij} - \delta_j) X_{i,j} - U_{i,j} \right].$$

where

- d_{ij} (km) denotes the distance of route (i,j);
- f_{ij} (€/km) denotes the fuel cost of route (i,j);
- t_{ij} (h) denotes the time needed to travel route (i,j);
- δ_j (h) denotes the expected amount of time needed to execution the task in position $j \in P$;
- g_j (€/h) denotes the profit of the photo booth associatet to task $j \in P$;
- q_i (€/day) denotes the cost of the technician for one day of work;
- d is the total number of days.

Set of constraints: adaptation from the multi-commodity flow

$$\min \; f(X,F,U) \tag{1}$$

s.t. $\sum_{j \in V} X_{i,j} \le 1, \quad \forall i \in V,$ (*each position is left at most once*) (2)

$\sum_{j \in V} (X_{i,j} - X_{j,i}) = 0, \quad \forall i \in V,$ (*each position is left if it is entered*) (3)

$\{(i,j) \in E \mid X_{i,j} = 1\}$ contains no circuit with zero depots (subtour elimination constraints) (4)

$\{(i,j) \in E \mid X_{i,j} = 1\}$ contains no circuit with two or more depots (path elimination constraints) (5)

$\{(i,j) \in E \mid U_{i,j} > 0\}$ all working days and time-windows are met (timing constraints) (6)

Set of constraints: subtour/path elimintation

When a position $k \in P$ is visited, a commodity (think about a receipt) is collected

Subtour elimination constraints

Commodities can flow through edges in a tour $(\forall j \in V,\ \forall i,k \in P \cup D,\ :\ i \neq j)$:

$$0 \leq F_{i,j,k} \leq X_{i,j}.$$

Commodities must leave their position if in a tour $(\forall k \in P \cup D)$:

$$\sum_{j \in V} (F_{k,j,k} - X_{k,j}) = 0.$$

Commodities leave a position iff they visited it $(\forall i,k \in P \cup D)$:

$$\sum_{j \in P \cup D} F_{j,i,k} = \sum_{j \in V} F_{i,j,k}.$$

Path elimination constraints

Commodities must return to a depot if in a tour $(\forall k \in P \cup D)$:

$$\sum_{j \in P \cup D} \sum_{i \in S} F_{j,i,k} \geq \sum_{j \in V} X_{j,k}.$$

Commodities first visited by a technician must return to their depots $(\forall i \in S,\ \forall k \in P \cup D)$:

$$\sum_{j \in P \cup D} F_{j,i,k} \geq X_{i,k}.$$

Set of constraints: timing constraints

Technicians must leave/return their depots within the horizon time ($\forall i \in S$, $\forall k \in P \cup D$):

$$T_i^s X_{i,k} \le U_{i,k} \le (T_i^e + 24(d-1))X_{i,k},$$
$$T_i^s X_{i,k} \le U_{k,i} \le (T_i^e + 24(d-1) - t_{k,i})X_{i,k}.$$

Technicians must leave/return their depots every night within the working time ($\forall i \in V$, $\forall k \in D$):

$$(T_k^s - t_{i,k})X_{i,k} \le U_{i,k},$$
$$(T_k^e - t_{i,k} - \delta_k)X_{i,k} \ge U_{i,k}.$$

Technicians must not work on a free day $\ell^\star \in \{1, \dots, d\}$:

$$X_{p,q} = 1, \quad \text{where} \quad \begin{cases} p &= (\ell^\star - 1)s + i, \\ q &= (\ell^\star \,\%\, d)s + i, \end{cases}$$

where $a \,\%\, b$ returns the reminder after a/b.

Tasks must be executed within their time windows ($\forall i \in V$, $\forall k \in P$):

$$(T_k^s - t_{i,k})X_{i,k} + 24 \sum_{\ell \in D} F_{i,k,\ell} \le U_{i,k},$$
$$(T_k^e - t_{i,k} - \delta_k)X_{i,k} + 24 \sum_{\ell \in D} F_{i,k,\ell} \ge U_{i,k}.$$

Tasks must be executed after the previous task's position has been reached and the task has been executed ($\forall k in P \cup D$):

$$\sum_{j \in V} U_{k,j} \ge \sum_{j \in V} \left(U_{j,k} + (t_{j,k} + \delta_k)X_{j,k}\right).$$

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Gossip-based communication scheme

⇒ **Gossiping**:

- Nodes in a network are said to communicate through gossiping if, at any time, any node may communicate only to one random neighbor at a time;
- Gossiping is a way to spread information in a decentralized and asynchronous manner;

⇒ In our proposed heuristic optimization algorithm gossiping is used to solve **local optimization problems between neighboring agents**:

- Each local optimization has a significantly smaller computational complexity with respect to the whole workforce problem;
- The heuristic can be used both offline (to find a sub-optimal initial solution) and online (to deal with unexpected events and real-time changes in the problem);
- In the online mode, the heuristic deals with:
  - Delays experienced by the technicians (e.g., traffic jams or difficulties in executing the tasks);
  - Sudden unavailability of agents (e.g., illness);
  - New tasks to be executed or new technicians available for executing tasks.

The Delaunay triangulation

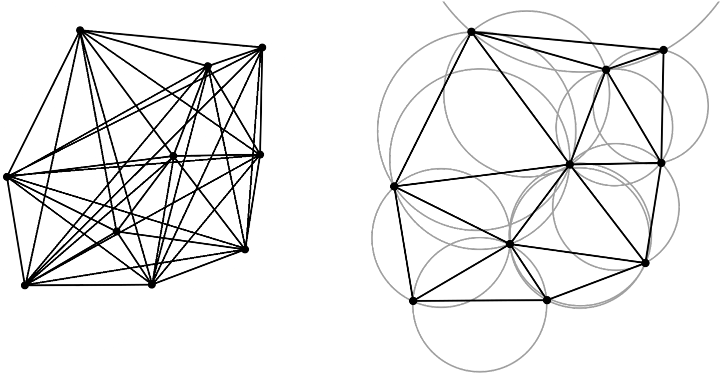


Fig. 1: Example of two graphs where the nodes represent the technicians depots: full graph (left) and Delaunay triangulation (right)

Heuristic gossip-based optimization algorithm - Part I

Input: The sets of technicians S, days D, and tasks P; The initial Delaunay graph $G_D = (S, E_D)$;
The number $m \in \mathbb{N}$ of unassigned tasks to be included in the local optimizations.

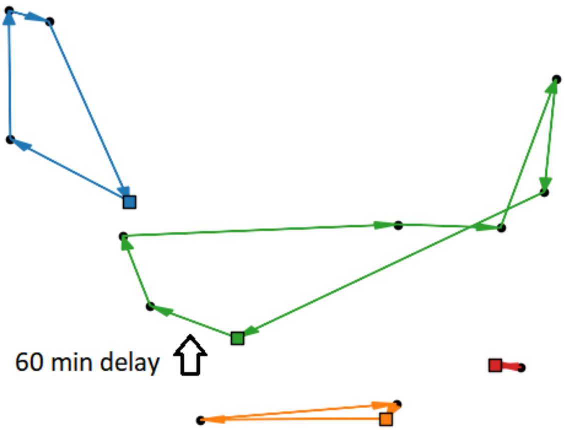
Initialize: Mark all edges as active $E_A = E_D$

1: **repeat** indefinitely steps 2-21:
2:     **while** $E_A = \varnothing$ **do**
3:         **if** technician $i^\star$ is late or changes task
4:             Mark edges of $i^\star$ as active: $E_A = E_A \cup \{(i,j) \in E_D \,|\, i = i^\star \text{ or } j = i^\star\}$
5:         **end if**
6:     **end while**
7:     Update technicians positions based on the tasks they are currently executing
8:     Save old edges $E_D^- = E_D$ and reconstruct the Delaunay Graph $\mathcal{G}_D = (S, E_D)$
9:     Mark old edges as inactive $E_A = E_A \cap E_D$
10:     Mark new edges as active $E_A = E_A \cup (E_D \setminus E_D^-)$

Heuristic gossip-based optimization algorithm - Part II

Pick an edge $(i^\star, j^\star)$ **at random from** E_A **and solve the local optimization problem where**:
(i) only technicians $i^\star$ and $j^\star$ are considered
(ii) only tasks currently assigned to technicians $i^\star$ or $j^\star$ are considered, along with $m \in \mathbb{N}$ randomly picked unassigned tasks in P
(iii) the scheduling of the current and previous tasks must remain unchanged
if the new solution improves the objective function
  if agent $i^\star$ changes assignment
    Mark its incident edges as active $E_A = E_A \cup \{(i,j) \in E_D \,|\, i = i^\star \text{ or } j = i^\star\}$
  end if
  if agent $j^\star$ changes assignment
    Mark its incident edges as active $E_A = E_A \cup \{(i,j) \in E_D \,|\, i = j^\star \text{ or } j = j^\star\}$
  end if
  return the new assignment, schedule, and routing for technicians $i^\star$ and $j^\star$
end if
Mark edge $(i^\star, j^\star)$ as inactive $E_A = E_A \setminus \{(i^\star, j^\star)\}$

A simple example with 4 employees and 12 tasks (online)



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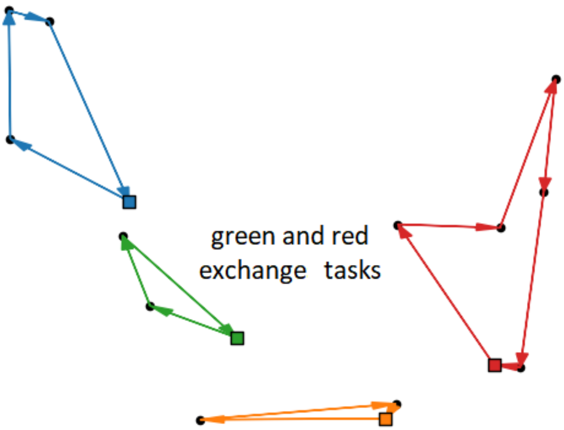

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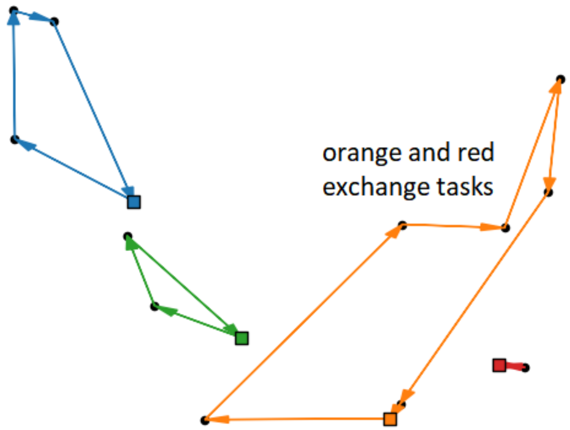

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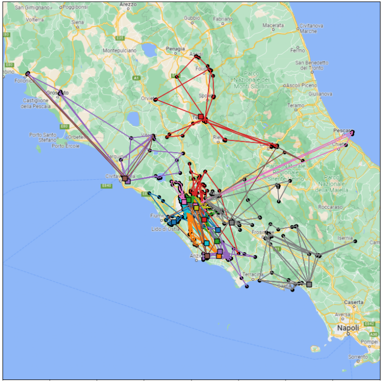
We consider a real-world example drawn from data provided by DEDEM S.p.A., an international company with a long history in the passport photo sector, starting in 1962 with the installation of the first photo booth machine in Rome, Italy. The problem under study consists of:

- **932 tasks** located in 5 italian regions (Lazio, Campania, Toscana, Umbria, Abruzzo)
- **16 technicians**
- **5 days of work** (one working week)

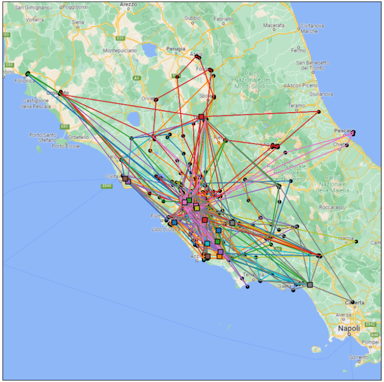
Parameters:

- Technicians start working at $7:00$ AM ($T_i^s = 7$) and finish working at $6:30$ PM ($T_i^e = 18.5$), for all $i \in S$;
- The cost of technicians for one day of work is € $50,00$ ($q_i = 50$ for all $i \in S$);
- Tasks may be executed at any time ($T^s = 7$ and $T_i^e = 18.5$ for all $p \in P$);
- The duration of the tasks ranges from 1 minute to 5 hours ($d_p \in [0.02, 5]$ for all $p \in P$), while the average duration is about 22 minutes;
- The profit of the tasks ranges from € 0.50 to € 11.40 per hour ($g_p \in [0.5, 11.4]$ for all $p \in P$), while the average profit is about € 1.80;
- The area containing all tasks is contained within a squared area of 250 km side length;
- Travel times have been computed by means of Google Maps APIs, while the fuel cost has been considered fixed for all roads and equal to about € 0.10 per kilometer ($f_{ij} = 0.10$ for any $i, j \in V$).

Offline solution



Real routes/scheduling provided by DEDEM



Solution provided by our heuristic algorithm

Task assigned: 814
Expected profit: $\sim 281k$ €

Task assigned: 832 ($\sim 2\%$ more)
Expected profit: $\sim 326k$ € ($\sim 15\%$ more)

Online solution

We consider the scenario in which the following unfortunate and unexpected event takes place:

- One of the technician is not available at the beginning of the first day!

If not action is taken, then:

- 19 tasks won't be executed
- The expected profit loss amounts to $\sim 11k$ €

Running our gossip-based heuristic one achieves results in:

- 17 out of the 19 tasks are re-assigned to other technicians in real-time
- The profit loss drops at $\sim 1k$ € (90% of the expected loss has been avoided)

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The **main contributions** are:

- A MILP formulation has been provided to solve a joint assignment/routing/scheduling problem inspired by the real case study of DEDEM S.p.A.;
- An heuristic gossip-based solution has been provided to solve the problem both offline and online to deal with dynamic changes in the problem formulation;

The **future directions** are:

- Include tasks with fixed profit and deadlines
- Derive a constant factor approximation
- Carry out Monte Carlo simulations with delays

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Thank you for your attention!

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